THE ACTION OF A STAMP ON AN ELASTIC ANISOTROPIC HALF-SPACE

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Studies dealing with the problem of action of a stamp on an isotropic or transversely isotropic half-space under the condition that the boundary of the half-space is a plane of elastic symmetry and that the stamp presses in the direction of the axis of elastic symmetry and has axisymmetric shape are generally known. However, only isolated papers [1, 5] or none at all are devoted to the action of a stamp on an anisotropic half-space in the more general case of anisotropy or the case of transversely isotropic body under the condition that the boundary of the half-space is not a plane of elastic symmetry.

It is shown below that the pressure distribution for a stamp with circular or elliptical plan form remains the same when the stamp acts on an anisotropic (orthotropic) half-space as it was for its action on an isotropic half-space. The method is based on the construction of a complex loading function corresponding to the given loading.

Formulas are presented for the impression of stamps of different shapes for the orthotropic body and the transversely isotropic medium. In the last case it was assumed that the boundary plane contains the axis of elastic symmetry. The case of off-center action of the compression force is examined. Through construction and utilization of the complex loading function the possibility of expanding the method to the case of an isotropic medium is indicated.

1. Orthotropic half-space under the action of a normal load. Equations of elastic equilibrium of an orthotropic body have the following solution [2]:

$$u(x, y, z) = \int_0^{2\pi} \operatorname{Re}\left[\sum_{k=1}^{3} \alpha\nu_k \Delta_k^{(1)} \omega_k(\Omega_k)\right] d\theta$$
$$v(x, y, z) = \int_0^{2\pi} \operatorname{Re}\left[\sum_{k=1}^{3} \beta\nu_k \Delta_k^{(2)} \omega_k(\Omega_k)\right] d\theta$$
$$w(x, y, z) = \int_0^{2\pi} \operatorname{Re}\left[\sum_{k=1}^{3} \Delta_k^{(3)} \omega_k(\Omega_k)\right] d\theta \qquad (1.1)$$

Here ν_k are the roots of the equation

$$\begin{vmatrix} A^* & (N+H)\alpha\beta & (M+G)\alpha\nu \\ (N+H)\alpha\beta & B^* & (L+F)\beta\nu \\ (M+G)\alpha\nu & (L+F)\beta\nu & C^* \end{vmatrix} = 0 \qquad (1.2)$$

$$A^* = A\alpha^2 + N\beta^2 + M\nu^2, \qquad B^* = N\alpha^2 + B\beta^2 + L\nu^2$$
$$C^* = M\alpha^2 + B\beta^2 + C\nu^2, \qquad \alpha = \cos\theta, \qquad \beta = \sin\theta$$

Functions ω_k, which depend on the argument $\Omega_k = x\alpha + y\beta + \nu_k z$, are arbitrary, $\alpha\nu_k\Delta_k^{(1)}$, $\beta\nu_k\Delta_k^{(2)}$ and $\Delta_k^{(3)}$ are minors of the determinant in the left side of Eq. (1.2), corresponding to elements of the third row and the root ν_k; B, C, A... are elastic constants. The pairwise complex conjugate roots ν_k depend only on α^2 and β^2, which follows from (1.2).

If on the boundary of the half-space $z \geqslant 0$ only the normal axisymmetric load is acting, then the functions ω_k, which are analytical in the upper half-plane and vanish at infinity, are found from equations which result from the boundary conditions

$$\sum_{k=1}^{3} \nu_k [G\alpha^2\Delta_k^{(1)} + F\beta^2\Delta_k^{(2)} + C\Delta_k^{(3)}]\, \omega_k = \Psi^+$$

$$\sum_{k=1}^{3} (\nu_k\Delta_k^{(j)} + \Delta_k^{(3)})\, \omega_k = 0 \qquad (j = 1,\ 2) \tag{1.3}$$

Here Ψ^+ is the boundary value of the function $\Psi(\Omega)$ determined by the following equations:

$$\frac{d\Psi}{d\Omega} = \frac{1}{\pi i}\int_{-\infty}^{\infty} \frac{\Phi(\tau)\, d\tau}{\tau - \Omega}, \qquad \Phi(\tau) = \frac{1}{2\pi}\frac{d}{d\tau}\int_0^{\tau} \frac{\sigma_z \rho d\rho}{\sqrt{\tau^2 - \rho^2}} \tag{1.4}$$

The function $\Psi(\Omega)$ vanishing at infinity, we call the complex loading function corresponding to a given loading on the boundary of the half-space.

In all subsequent investigations it is found directly by means of analytical extension.

If Δ_k are minors of determinant Δ_0 of the system of Eq. (1.3), then

$$\omega_k(\Omega_k) = \frac{\Delta_k}{\Delta_0}\Psi(\Omega_k) \qquad (k = 1,\ 2,\ 3) \tag{1.5}$$

Substituting this into (1.1) we obtain the equations for elastic displacements and, in particular

$$w(x,\ y,\ z) = \int_0^{2\pi} \sum_{k=1}^{3} \mathrm{Re}\, \frac{\Delta_k^{(3)}\Delta_k}{\Delta_0}\Psi(\Omega_k)\, d\theta \tag{1.6}$$

2. Action of a circular flat stamp on a orthotropic half-space.

Let a normal load of the following form act on an orthotropic half-space

$$\sigma_z = \frac{-P}{2\pi R_0 (R_0^2 - \rho^2)^{1/2}} \qquad (\rho < R_0)\ \sigma_z = 0 \qquad (\rho > R_0) \tag{2.1}$$

We have

$$\Phi(\xi) = \frac{P}{8\pi^2 R_0}\frac{d}{d\xi}\ln\frac{|\xi - R_0|}{|\xi + R_0|} \qquad (\xi = x\alpha + y\beta) \tag{2.2}$$

From this

$$\Psi(\Omega) = \frac{P}{8\pi^2 R_0}\ln\frac{\Omega - R_0}{\Omega + R_0} \tag{2.3}$$

The principal values for the logarithms are taken here. Correspondingly we obtain

$$\Psi^+ = \frac{P}{8\pi^2 R_0}\left[\ln\frac{|\xi - R_0|}{|\xi + R_0|} + \pi i\right] \qquad (|\xi| < R_0)$$

$$\Psi^+ = \frac{P}{8\pi^2 R_0}\ln\frac{|\xi - R_0|}{|\xi + R_0|} \qquad (|\xi| > R_0) \tag{2.4}$$

From (1.6) we derive

$$w(x,\ y,\ 0) = \int_0^{2\pi} \sum_{k=1}^{3} \mathrm{Re}\, \frac{\Delta_k^{(3)}\Delta_k}{\Delta_0}\Psi^+ d\theta \tag{2.5}$$

We further have

$$x = \rho\cos\varphi, \qquad y = \rho\sin\varphi, \qquad \xi = \rho\cos(\theta - \varphi)$$

If $\rho < R_0$, then $|\xi| < R_0$. Therefore, according to (2.4), in points on the half-space boundary where $\rho < R_0$, we obtain

$$w(x, y, 0) = \frac{P}{2\pi R_0} \int_0^{1/2\pi} \sum_{k=1}^{3} \operatorname{Re} \frac{\Delta_k^{(3)}\Delta_k}{\Delta_0} i d\theta \tag{2.6}$$

It is taken into account here that integrals of the form

$$\int_0^{2\pi} E(\alpha^2, \beta^2) \ln \frac{|\xi - R_0|}{|\xi + R_0|} d\theta$$

are equal to zero and that the integrand in (2.6) depends only on α^2 and β^2. In this manner for points of the half-space where $\rho < R_0$ elastic displacements normal to the boundary turn out to be constant. It follows from this that solution (1.1) which corresponds to load (2.1) is the solution of the problem of pressure on an orthotripic half-space by a smooth circular flat stamp with a radius R_0. The stamp is loaded in the center by a force P. Equation (2.6) determines the impression of the stamp.

We note that for all real orthotropic bodies the integral in the right side of Eq. (2.6) is positive. In the opposite case we would have a result contradictory to the theorem of existence and uniqueness of the solution for the formulated problem.

A direct proof of the positiveness of the mentioned integral can easily be carried out for an orthotropic body of a particular shape when the elastic constants are connected through the following additional relationships

$$B = A, \quad G = F, \quad M = L \tag{2.7}$$

and also the inequalities

$$HC - F^2 > 0, \qquad A > H \tag{2.8}$$

which are satisfied for all real bodies of the class (2.7), presented in [3]. However, we shall not dwell on this.

Below, in the case of transversely isotropic body the positiveness of the integral which determines the impression of the stamp is established directly.

Equation (2.5) makes it possible to find the normal displacements of points of the boundary which do not lie inside the circle $\rho \leqslant R_0$. Here the integrand in (2.5) will be different from zero for a set of values θ, which satisfy the inequalities

$$\rho \cos(\theta - \varphi) < R_0, \qquad \rho > R_0 \tag{2.9}$$

3. Action of a flat stamp with an elliptical plan form. Let E be the region of the plane $z = 0$ which is bounded by the ellipse with half-axes a and b. Let CE be its complement to the full plane.

The normal stresses in points $M(x, y, 0)$ of the boundary of the half-space are given in the form

$$\sigma_z = \frac{-P}{2\pi\sqrt{ab}\,(1 - x^2/a^2 - y^2/b^2)^{1/2}} \qquad (M \in E)$$

$$\sigma_z = 0 \qquad (M \in CE) \tag{3.1}$$

Setting

$$\sigma_z(x, y, 0) = \int_0^{2\pi} f\left(\frac{x\alpha + y\beta}{\Delta}\right) \frac{d\theta}{\Delta^2}, \qquad \Delta^2 = (l_2^2\alpha^2 + l_1^2\beta^2) \qquad \begin{pmatrix} l_1^2 = b/a \\ l_2^2 = a/b \end{pmatrix} \tag{3.2}$$

we obtain

$$\sigma_z(r_1, 0) = \int_0^{2\pi} f(x_1\alpha_1 + y_1\beta_1)\, d\theta_1 \qquad \begin{pmatrix} x_1 = l_1 x, \ y_1 = l_2 y \\ r_1^2 = x_1^2 + y_1^2 \end{pmatrix} \tag{3.3}$$

$$\alpha_1 = \cos\theta_1 = l_2\alpha\Delta^{-1}, \qquad \beta_1 = \sin\theta_1 = l_1\beta\Delta^{-1}, \qquad d\theta_1 = \Delta^{-2} d\theta$$

Here we take advantage of the fact that if $\theta \in (0.2\pi)$, then also $\theta_1 \in (0,2\,\pi)$, and vice versa. In this manner the solving function corresponding to load (3.1) has the form

$$\Psi(\Omega_k) = \frac{P}{8\pi^3 \sqrt{ab}} \ln \frac{\Omega_k - \sqrt{ab}}{\Omega_k + \sqrt{ab}} \tag{3.4}$$

i.e. it is obtained from (2.3) through replacement of R_0 by $\sqrt{ab}$. In this connection, in Eqs. (1.1)–(1.3), (2.5) and (2.6), it is necessary to replace α and β by $\alpha\Delta^{-1}$ and $\beta\Delta^{-1}$ and to set $\Omega_k = (x\alpha + y\beta + \nu_k z)\,\Delta^{-1}$.

Correspondingly we obtain

$$w(x,\, y,\, 0) = \frac{P}{2\pi \sqrt{ab}} \int_0^{2\pi} \sum_{k=1}^{3} \operatorname{Re} \frac{\Delta_k^{(3)} \Delta_k}{\Delta_0}\, i\, \frac{d\mathbb{C}}{\Delta} \tag{3.5}$$

4. Transversely isotropic body. Here we have simpler results. If the axis z is the axis of elastic symmetry, then, setting

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}, \qquad v = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}, \qquad w = \frac{\partial\chi}{\partial z} \tag{4.1}$$

we write the equilibrium equation in the form [2]

$$\begin{gathered} A\nabla\varphi + L\frac{\partial^2\varphi}{\partial z^2} + (L+F)\frac{\partial^2\chi}{\partial z^2} = 0, \qquad N\nabla\psi + L\frac{\partial^2\psi}{\partial z^2} = 0 \\ (L+F)\nabla\varphi + L\nabla\chi + C\frac{\partial^2\chi}{\partial z^2} = 0, \qquad \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{gathered} \tag{4.2}$$

Taking the boundary of the half-space to be the plane $y = 0$, we construct the solution (4.2) in the form

$$\varphi = \operatorname{Re} \int_0^{2\pi} \sum_{k=1}^{2} \varphi_k(\Omega_k)\, d\theta, \qquad \chi = \operatorname{Re} \int_0^{2\pi} \sum_{k=1}^{2} \chi_k(\Omega_k)\, d\theta, \tag{4.3}$$

$$\psi = \operatorname{Re} \int_0^{2\pi} \psi_1(\Omega_3)\, d\theta \qquad \Omega_p = \xi + \nu_p y, \qquad \nu_p = i\lambda_p = \sqrt{\alpha^2 + \beta^2\gamma_p^{-2}} \qquad (p = 1,\, 2)$$

We obtain

$$\xi = \alpha x + \beta z, \qquad \alpha = \cos\theta, \qquad \beta = \sin\theta$$

$$\begin{gathered} \varphi_k' = \sum_{k=1}^{2} (\gamma_k^2 C - L)\, \omega_k(\Omega_k), \qquad \chi_k' = (L+F) \sum_{k=1}^{2} \omega_k(\Omega_k) \\ \psi_1' = \omega_3(\Omega_3), \qquad \varphi_k' = d\varphi_k / d\Omega_k, \ldots \end{gathered} \tag{4.4}$$

Here $\gamma_3^2 = N/L$, and γ_k are roots of the equation

$$\begin{vmatrix} A - \gamma^2 L & -(L+F)\gamma^2 \\ L+F & L - \gamma^2 C \end{vmatrix} = 0 \tag{4.5}$$

In this connection γ_k^2 are real if $\sqrt{AC} > 2L + F$ and one of the roots (which will be designated γ_1) is greater than unity.

On the boundary of the half-space we have for σ_y the conditions (2.1). Here the tangential stresses τ_{yx} and τ_{yz} are equal to zero.

The determinant Δ_0 in the given case has the form

$$\Delta_0 = i \det |\, a_{kl} \,| \qquad (k = 1,\, 2,\, 3) \tag{4.6}$$

Its elements have the following expressions:

$$\begin{gathered} a_{1l} = L[(\gamma_l^2 C + F)\beta^2 + 2(\gamma_l^2 C - L)\alpha^2 \quad (l = 1,\, 2), \qquad a_{13} = 2N\alpha\lambda_3 \\ a_{2l} = 2(\gamma_l^2 C - L)\lambda_l\alpha \quad (l = 1,\, 2), \qquad a_{23} = \beta^2\gamma_3^{-2} + \alpha^2 \\ a_{3l} = (\gamma_l^2 C + F)\lambda_l \quad (l = 1,\, 2), \qquad a_{33} = \alpha, \end{gathered} \tag{4.7}$$

We obtain

$$\Delta_0 = i\,(L+F)\,C\,(\gamma_2^2-\gamma_1^2)\,\beta^2\Delta_1(\alpha,\ \beta) \tag{4.8}$$

Here

$$\Delta_1(\alpha,\beta)=\frac{4\alpha^2 L\lambda_2\lambda_3(\gamma_1^2-\gamma_3^2)}{\gamma_1^2(\lambda_1+\lambda_2)}+$$
$$+\frac{L(AC-F\cdot)\beta^2+4LN(\gamma_2^2C+F)\alpha^2\beta^2+4N^2(\gamma_1^2C-L)\alpha^4}{NA(\lambda_1+\lambda_2)} \tag{4.9}$$

If $\sqrt{AC}>2L+F$ and $N\leqslant L$, then $\Delta_1(\alpha,\ \beta)>0$ for all real α and β.

We obtain

$$v(x,\ 0,\ z)=-4\int_0^{1/_2\pi}\frac{\lambda_1\lambda_2}{\Delta_1(\alpha,\beta)}\operatorname{Re} i\Psi^+\,d\theta \tag{4.10}$$

The settling of the stamp is determined by the following equation in accordance with (2.4)

$$v(x,\ 0,\ z)=\frac{P}{2\pi R_0}\int_0^{1/_2\pi}\frac{\lambda_1\lambda_2}{\Delta_1(\alpha,\beta)}\,d\theta \tag{4.11}$$

For a flat stamp with elliptical plan form we obtain correspondingly

$$v(x,\ 0,\ z)=\frac{P}{2\pi\sqrt{ab}}\int_0^{1/_2\pi}\frac{\lambda_1\lambda_2}{\Delta_1(\alpha,\beta)}\frac{d\theta}{\Delta} \tag{4.12}$$

If the medium is isotropic, then

$$C=A=\lambda+2\mu,\quad L=N=\mu,\quad F=\lambda,\quad \gamma_1=\gamma_2=\gamma_3=1 \tag{4.13}$$

Substituting into (4.12) we derive the known result [4]

$$v(x,\ 0,\ z)=\frac{(\lambda+2\mu)P}{4\pi\mu(\lambda+\mu)ab}\int_0^{1/_2\pi}\frac{d\theta}{\sqrt{\alpha^2/a^2+\beta^2/b^2}} \tag{4.14}$$

5. A stamp bounded by the surface of an elliptical paraboloid. Let the normal stresses in points M of the boundary be distributed according to the following relationships

$$\sigma_y(x,\ 0,\ z)=\frac{3P}{2\pi ab}\left(1-\frac{x^2}{a^2}-\frac{z^2}{b^2}\right)^{1/2}\qquad (M\in E)$$
$$\sigma_y(x,\ 0,\ z)=0\qquad (M\in CE) \tag{5.1}$$

In the given case it is easy to show that

$$\Psi(\Omega_k)=\frac{3P}{8\pi(ab)^{3/2}}\left[-\sqrt{ab}\,\Omega_k+\frac{ab-\Omega_k^2}{2}\ln\frac{\Omega_k-\sqrt{ab}}{\Omega_k+\sqrt{ab}}\right] \tag{5.2}$$

Here

$$\Omega_k=(\alpha x+i\lambda_k y+\beta z)\cdot\Delta^{-1}$$

For $y=0$, taking into account the boundary value of function Ψ when the selection of branches of logarithms is made as indicated above, we obtain

$$v(x,\ 0,\ z)=\frac{3P}{16\pi(ab)^{3/2}}\int_0^{2\pi}\frac{\lambda_1\lambda_2}{\Delta_1(\alpha,\beta)}\left[ab-\frac{(\alpha x+\beta z)^2}{\Delta^2}\right]\frac{d\theta}{\Delta} \tag{5.3}$$

From this

$$v(x,\ 0,\ z)=\delta-J_1x^2-J_2z^2,\qquad \delta=\frac{3P}{4\pi\sqrt{ab}}\int_0^{1/_2\pi}\frac{\lambda_1\lambda_2}{\Delta_1(\alpha,\beta)}\frac{d\theta}{\Delta}$$
$$J_l=\frac{3P}{4\pi(ab)^{3/2}}\int_0^{1/_2\pi}\frac{\lambda_1\lambda_2\alpha_l^2}{\Delta_1(\alpha,\beta)}\frac{d\theta}{\Delta}\qquad\left(\begin{matrix}\alpha_1=\alpha,\ \alpha_2=\beta\\ l=1,\,2\end{matrix}\right) \tag{5.4}$$

The term containing the product xy drops out because the corresponding integral becomes zero. Considering the area of contact between the stamp and the half-space to be sufficiently small, we obtain

$$J_1 = R_1^{-1}, \qquad J_2 = R_2^{-1} \tag{5.5}$$

Here R_1 and R_2 are the principal radii of curvature of the surface bounding the stamp at the point M (0, 0, 0). From (5.5) we find l_1 and $l_2 = l_1^{-1}$, and subsequently a, b and δ.

We note that in the case under examination a contact area of elliptical shape corresponds to a stamp with a circular plan form because for $a = b$ the values of J_1 and J_2 are different.

6. Pressure of a stamp with circular plan form with off-center application of pressing force. Normal stresses in the circle $\rho < R_0$ of the boundary of the half-space $y = 0$ are given in the form

$$\sigma_y = -P/2\pi R_0^2 \, [1 + 3(x_0 x + z_0 z) R_0^{-2}]. \, (R_0^2 - \rho^2)^{-1/2} \tag{6.1}$$

Let us construct the complex loading function corresponding to the load (6.1). We set

$$\frac{x}{(R_0^2 - \rho^2)^{1/2}} = \int_0^{2\pi} \alpha f(\xi)\, d\theta, \qquad \xi = \alpha x + \beta z \qquad \begin{pmatrix} \alpha = \cos\theta \\ \beta = \sin\theta \end{pmatrix} \tag{6.2}$$

It is easy to establish that

$$\int_0^{2\pi} \alpha f(\xi)\, d\theta = \cos\varphi \int_0^{2\pi} \alpha f(\rho\alpha)\, d\theta \qquad \begin{pmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{pmatrix} \tag{6.3}$$

For $f(\xi)$ we obtain therefore the following equation:

$$\frac{\rho}{(R_0^2 - \rho^2)^{1/2}} = \int_0^{2\pi} \xi f(\xi)\, d\theta \tag{6.4}$$

which reduces to Abel's equation. Its solution is written in the following form:

$$f(\xi) = \frac{1}{4\pi} \frac{d}{d\xi} \xi \ln \left| \frac{\xi - R_0}{\xi + R_0} \right| \tag{6.5}$$

By analogy we have

$$\frac{z}{(R_0^2 - \rho^2)^{1/2}} = \int_0^{2\pi} \beta f(\xi)\, d\theta \tag{6.6}$$

In this manner, taking into account the representation of function $(R_0^2 - \rho^2)^{-1/2}$, we obtain the transformation of (6.1) in the form

$$\Phi(\xi) = \frac{P}{8\pi R_0} \frac{d}{d\xi} \left\{ \left[1 + \frac{3(x_0\alpha + z_0\beta)\xi}{R_0^2} \right] \ln \left| \frac{\xi - R_0}{\xi + R_0} \right| \right\} \tag{6.7}$$

Without violating (6.7) and with the requirement of disappearance of $\Psi(\Omega)$ at infinity, we derive

$$\Psi(\Omega) = \frac{P}{8\pi R_0} \left\{ \left[1 + \frac{3(x_0\alpha + z_0\beta)}{R_0^2} \Omega \right] \ln \frac{\Omega - R_0}{\Omega + R_0} + \frac{6(x_0\alpha + z_0\beta)}{R_0} \right\} \tag{6.8}$$

It is easy to verify that the second term in parentheses has no effect on the settling of the stamp. The selection of branches of logarithms was indicated above. Substituting into (4.10) we obtain for $y = 0$ and $\rho < R_0$

$$v(x, 0, z) = \frac{P}{8\pi R_0} \int_0^{2\pi} \frac{\lambda_1\lambda_2}{\Delta_1(\alpha, \beta)} \left[1 + \frac{3(x_0\alpha + z_0\beta)}{R_0^2} \xi \right] d\theta \tag{6.9}$$

or (since the integral containing the product $\alpha\beta$ is equal to zero),

$$v(x, 0, z) = \frac{P}{2\pi R_0} \int_0^{1/2\pi} \frac{\lambda_1\lambda_2}{\Delta_1(\alpha, \beta)} \left[1 + \frac{3(x_0 x\alpha^2 + z_0 z\beta^2)}{R_0^2}\right] d\theta \tag{6.10}$$

It follows from (6.10) that the loading function (6.8) realizes the solution of the problem of action of a circular flat stamp on an anisotropic half-space for the case where the force P is applied off-center.

If a flat stamp with elliptical plan form is loaded in the same manner by a force P, then using the same reasoning as above, we obtain correspondingly

$$\Psi(\Omega_k) = \frac{P}{8\pi\sqrt{ab}} \left\{\left[1 + \frac{3(x_0\alpha + z_0\beta)}{ab\Delta}\Omega_k\right] \ln\frac{\Omega_k - \sqrt{ab}}{\Omega_k + \sqrt{ab}} + \right.$$
$$\left. + \frac{6(x_0\alpha + z_0\beta)}{\sqrt{ab}}\right\}, \qquad \Omega_k = (x\alpha + z\beta + i\lambda_k y)\,\Delta^{-1} \tag{6.11}$$

Function (6.11) corresponds to the load

$$\sigma_y(x, 0, z) = -\frac{P}{2\pi\sqrt{ab}}\left(1 + \frac{3x_0 x}{a^2} + \frac{3z_0 z}{b^2}\right)\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{-1/2} \tag{6.12}$$

The expression in parentheses containing x_0 and z_0 must be positive. This places a limitation on the location of the point at which the force P is applied. The displacements of points under the stamp are found from the following formula:

$$v(x, 0, z) = \frac{P}{2\pi\sqrt{ab}} \int_0^{1/2\pi} \frac{\lambda_1\lambda_2}{\Delta_1(\alpha, \beta)} \left[1 + \frac{3x_0 x\alpha^2 + 3z_0 z\beta^2}{ab\Delta^2}\right] \frac{d\theta}{\Delta} \tag{6.13}$$

Analogous results can be written for the orthotropic body.

Taking (4.13), we obtain the known results for the isotropic medium.

In all cases examined, the elastic displacements in the half-space disappear not slower than $(x^2 + y^2 + z^2)^{-1/2}$.

From the material presented above we conclude that at least for the orthotropic body and the transversely isotropic medium the pressure distribution under the stamp does not depend on the form of anisotropy in the problems which were examined. The dimensions of the pressure area (if they are not given) and the settling of the stamp depend on the form of anisotropy.

In conclusion we note the possibility of application of this method in the case of an isotropic body. As a preliminary it is necessary here to construct the solution (1.1). For this purpose it is sufficient to insert (1.5) into (1.1) and to go to the limit with utilization of (4.13).

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